# Lecture 30. Phylogeny methods, part 2 (Searching tree space) 

Joe Felsenstein
Department of Genome Sciences and Department of Biology

## All possible trees



Forming all 4-species trees by adding the next species in all possible places

## The number of rooted bifurcating trees:

$$
\begin{aligned}
& 1 \times 3 \times 5 \times 7 \times \ldots \times(2 n-3) \\
& =(2 n-3)!/\left((n-2)!2^{n-2}\right)
\end{aligned}
$$

## which is:

| species | number of trees |
| :---: | :---: |
| 1 | 1 |
| 2 | 1 |
| 3 | 3 |
| 4 | 15 |
| 5 | 105 |
| 6 | 945 |
| 7 | 10,395 |
| 8 | 135,135 |
| 9 | 2,027,025 |
| 10 | 34,459,425 |
| 11 | 654,729,075 |
| 12 | 13,749,310,575 |
| 13 | 316,234,143,225 |
| 14 | 7,905,853,580,625 |
| 15 | 213,458,046,676,875 |
| 16 | 6,190,283,353,629,375 |
| 17 | 191,898,783,962,510,625 |
| 18 | 6,332,659,870,762,850,625 |
| 19 | 221,643,095,476,699,771,875 |
| 20 | 8,200,794,532,637,891,559,375 |
| 30 | $4.9518 \times 10^{38}$ |
| 40 | $1.00985 \times 10^{57}$ |
| 50 | $2.75292 \times 10^{76}$ |

## Rooting an unrooted tree



## A global maximum is not easy to find


a subtree

is rearranged by dissolving the connections to an interior branch:

and reforming them in one of the two possible alternative ways:


Nearest-neighbor interchange (NNI) rearrangement of a tree (the triangles are subtrees)

## all 15 trees, connected by NNIs



## with parsimony scores



## Subtree pruning and regrafting (SPR) rearrangement



Add it in, attaching to one (*) of the other branches


## Tree bisection and reconnection (TBR) rearrangement



L
Here is that result:


## Greedy search by sequential addition



Greedy search by addition of species in a fixed order (A, B, C, D, E) in the best place each time.

## Goloboff's time-saving trick



Goloboff's economy in computing scores of rearranged trees

Star decomposition

"Star decomposition" search for best tree can happen in multiple ways

## Disk-covering


"Disk covering" - assembly of a tree from overlapping estimated subtrees

## Shortest Hamiltonian path problem



## Search tree for this problem



Lecture 30. Phylogeny methods, part 2 (Searching tree space) - p.17/2

Search tree of trees

same, with parsimony scores in place of trees


## Some references

Camin, J. H. and R. R. Sokal. 1965. A method for deducing branching sequences in phylogeny.
Evolution 19: 311-326. [Early parsimony paper includes rearrangement of trees]
Cavalli-Sforza, L. L. and A. W. F. Edwards. 1967. Phylogenetic analysis: models and estimation procedures. American Journal of Human Genetics 19: 233-257. also Evolution 21: 550-570.
[Includes counting and tree shapes]
Farris, J. S. 1970. Methods for computing Wagner trees. Systematic Zoology 19: 83-92. [Early parsimony algorithms paper is one of first to mention sequential addition strategy]
Felsenstein, J. 1978. The number of evolutionary trees. Systematic Zoology 27: 27-33. (Correction, vol. 30, p. 122, 1981) [Review of counting tip-labelled trees, recursion for counting multifurcating case]
Foulds, L. R. and R. L. Graham. 1982. The Steiner problem in phyloge ny is NP-complete. Advances in Applied Mathematics 3: 43-49. [Parsimony is NP-hard]
Graham, R. L. and L. R. Foulds. 1982. Unlikelihood that minimal phylogenies for a realistic biological study can be constructed in reasonable computat ional time. Mathematical Biosciences 60: 133-142. [ ... and more]
Hendy, M. D. and D. Penny. 1982. Branch and bound algorithms to determine minimal evolutionary trees. Mathematical Biosciences 60: 133-142 [Introduced branch-and-bound for phylogenies]

## continued

Huson, D., S. Nettles, L. Parida, T. Warnow, and S. Yooseph. 1998. The disk-covering method for tree reconstruction. pp. 62-75 in Proceedings of "Algorithms and Experiments" (ALEX98), Trento, Italy, Feb. 9-11, 1998, ed. R. Battiti and A. A. Bertossi. ["Disk-covering method" for long stringy trees]
Maddison, D. R. 1991. The discovery and importance of multiple islands of most-parsimonious trees. Systematic Zoology 40: 315-328. [Discusses heuristic search strategy involving ties, multiple starts]
Saitou, N., and M. Nei. 1987. The neighbor-joining method: a new method for reconstructing phylogenetic trees. Molecular Biology and Evolution 4: 406-425. [First mention of star-decomposition search for best trees, sort of]
Strimmer, K., and A. von Haeseler. 1996. Quartet puzzling: a quartet maximum likelihood method for reconstructing tree topologies. Molecular Biology and Evolution 13: 964-969. [Assembles trees out of quartets]
Swofford, D. L. and G. J. Olsen. 1990. Phylogeny reconstruction. Chapter 11, Pp. 411-501 in Molecular Systematics, ed. D. M. Hillis and C. Moritz. Sinauer Associates, Sunderland, Massachusetts. [Review that discusses strategies, names SPR and TBR rearrangement methods]
Waterman, M. S. and T. F. Smith. 1978. On the similarity of dendrograms. Journal of Theoretical Biology 73: 789-800. [Defines NNIs. Uses them to get a distance between trees.]

## How it was done

This projection produced

- using the prosper style in LaTeX,
- using Latex to make a .dvi file,
- using dvips to turn this into a Postscript file,
- using ps2pdf to mill it into a PDF file, and
- displaying the slides in Adobe Acrobat Reader.

Result: nice slides using freeware.

